NONLINEAR TIME SERIES MODELLING AND REGIME SHIFTS IN BITCOIN RETURNS: STAR-BASED INVESTIGATION

Abstract

This study aimed to examine the nonlinear behaviour and regime-switching dynamics in Bitcoin [SS2.1] returns using a Smooth Transition Autoregressive (STAR) model. The analysis was conducted on daily Bitcoin data from 2020 to 2025. Descriptive statistics revealed high volatility, extreme kurtosis, and negative skewness, indicating non-normal and heavy-tailed return distributions. To assess the suitability of nonlinear modelling, various diagnostic tests were applied. While the RESET and BDS tests did not indicate strong evidence of nonlinearity, the Terasvirta neural network test confirmed significant nonlinear structure, supporting the use of a STAR model.

The STAR model was estimated with two lags and a logistic transition function. Results showed a sharp regime switch around a threshold return of −3.3%, with significant autoregressive behaviour in the low-return regime. Model fit indicators, including a lower AIC compared to the linear AR model, confirmed improved performance. However, residual diagnostics using the Box-Ljung test suggested some autocorrelation, indicating the possible need for volatility modelling. Forecasting over a 10-day horizon showed a transition from fluctuating returns to a stable upward trend.

The findings highlight the asymmetric behaviour of Bitcoin returns across regimes and support the application of nonlinear models in crypto currency forecasting. Practically, the results offer insights for short-term trading strategies and risk management. The study contributes originality by integrating STAR modelling with volatility tests for Bitcoin, providing a comprehensive nonlinear framework for digital asset analysis.

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Introduction

The explosive growth of crypto currencies, particularly Bitcoin, has revolutionized global financial markets and attracted significant attention from investors, researchers, and policymakers. Bitcoin's decentralized nature, extreme volatility, and susceptibility to speculative behaviour distinguish it from traditional financial assets and pose unique challenges for modelling and forecasting its price movements. As the adoption of digital assets expands across institutional and retail investors, understanding the underlying structure of Bitcoin returns becomes increasingly critical for informed decision-making and risk management.

Despite extensive research on linear time series models in financial forecasting, several empirical studies have shown that crypto currency returns exhibit nonlinear behaviour, regime shifts, and volatility clustering—characteristics that conventional models often fail to capture. While models such as ARIMA and GARCH have been widely applied, they are limited in their ability to model asymmetric dynamics and smooth transitions between different market states.

This study addresses this gap by applying a Smooth Transition Autoregressive (STAR) model to examine the nonlinear structure of Bitcoin returns. Unlike threshold models with abrupt shifts, the STAR model accommodates gradual regime changes and captures asymmetric return behaviour more effectively. The uniqueness of this research lies in its comprehensive approach: combining descriptive analysis, linearity testing, and nonlinear modelling to uncover the structural dynamics of Bitcoin returns over a recent period (2020–2025).

Several recent studies have attempted to model cryptocurrency behaviour using advanced econometric and machine learning techniques. However, few have explicitly focused on smooth transition models such as STAR in the context of Bitcoin returns. This study extends existing literature by validating model choice through formal nonlinearity tests (e.g., Terasvirta test) and comparing the STAR model with its linear counterparts using AIC and residual diagnostics.

Given the current trend of growing institutional interest in cryptocurrencies, this research is timely and relevant. It provides valuable insights into the predictive structure of Bitcoin returns and supports the use of nonlinear frameworks for forecasting and risk analysis in volatile digital markets.

Literature Review

The financial time series literature has increasingly focused on nonlinear modelling techniques to better capture the complex behaviour of asset returns. Hamilton (1989) introduced the Markov Switching model to explain regime shifts in macroeconomic indicators, laying the foundation for nonlinear modelling. Teräsvirta (1994) formalized the Smooth Transition Autoregressive (STAR) model to detect gradual transitions between regimes in time series data.

Tsay (1989) demonstrated the importance of testing for nonlinearity using threshold models and emphasized the limitations of linear ARIMA models in financial series. Following this, Granger and Teräsvirta (1993) provided a detailed framework for modelling economic time series using nonlinear autoregressive models, including STAR and TAR variants.

In the context of financial markets, Franses and van Dijk (2000) showed that STAR models outperform linear models when returns exhibit asymmetries and regime changes. Lux (1998) also argued that nonlinearities are prevalent in speculative markets, supporting the need for regime-switching models in return forecasting.

More recent studies have applied these models to cryptocurrencies. Chu et al. (2017) used GARCH-type models to capture Bitcoin's volatility, finding strong evidence of heteroskedasticity. Bouri et al. (2019) confirmed nonlinear dependencies in crypto currency returns using chaos and complexity measures.

Phillip, Chan, and Peiris (2018) found that Bitcoin exhibits both heavy tails and time-varying volatility, necessitating more flexible models. Caporale, Gil-Alana, and Plastun (2018) explored long memory and structural breaks in crypto markets, suggesting regime-based models.

Balcilar et al. (2017) used non-parametric methods and confirmed that Bitcoin returns are affected by investor sentiment and regime switching. Katsiampa (2017) compared AR-GARCH models and found GARCH variants useful for modelling Bitcoin volatility, though nonlinear models were underutilized.

Kristjanpoller and Minutolo (2018) applied machine learning and nonlinear models to forecast Bitcoin returns, showing better performance than linear alternatives. Shahzad et al. (2021) investigated asymmetric spill overs and nonlinear dependence in the crypto–equity space, further highlighting the relevance of regime-based models.

Overall, while numerous studies have addressed Bitcoin's volatility and structural complexity, few have specifically employed STAR models. This study builds on these gaps by combining formal nonlinearity testing and STAR modelling to better understand the asymmetric return behaviour of Bitcoin.

Research Gap

While previous studies have widely applied GARCH-type models and machine learning techniques to analyse cryptocurrency volatility, limited attention has been given to smooth transition models such as STAR. Few studies have explicitly tested for nonlinearity before model selection, and even fewer have applied STAR models to Bitcoin returns with formal justification. This study fills that gap by combining nonlinearity testing and STAR modelling to capture asymmetric regime-switching behaviour in Bitcoin returns.

Research Methodology

This study is **quantitative** and **exploratory** in nature, aimed at analysing the nonlinear dynamics of Bitcoin returns through advanced time series modelling. The design adopts a **time-series analytical approach**, utilizing historical price data of Bitcoin collected from Yahoo Finance using the quantmod package in R.

Daily adjusted closing prices of Bitcoin (BTC-USD) were collected for the period from **January 1, 2020, to March 30, 2025**. The log returns were calculated to ensure stationarity and remove scale issues. The data was selected using **purposive sampling**, focusing on a high-volatility, high-adoption period to better capture nonlinear behaviour in returns.

The **Terasvirta Neural Network Test**, **RESET test**, and **BDS test** were applied to detect the presence of nonlinearity in the mean structure of the return series. Based on the test outcomes, the **Smooth Transition Autoregressive (STAR)** model was selected and estimated using the tsDyn package in R, which allows for flexible nonlinear modelling. The model was compared against a linear AR model using AIC, and its residuals were assessed using the Box-Ljung test. Short-term forecasting was conducted, and performance was evaluated using RMSE and MAPE.

The use of **R software** was justified due to its powerful time series packages (quantmod, tsDyn, tseries, forecast, and e1071) and reproducibility in academic research. Confidence intervals for forecasts were manually derived from STAR residual variance.

Hypotheses of the Study

- **H_{01}**: There is no significant nonlinearity in Bitcoin return dynamics.
- **H_{11}**: Bitcoin return dynamics exhibit significant nonlinearity.
- **H_{02}**: The STAR model does not outperform the linear AR model in modelling Bitcoin returns.
- **H_{12}**: The STAR model provides a significantly better fit than the linear AR model.

Table-1 : Descriptive statistics

Metric	Value	Metric	Value
Observations (n)	1915	Minimum	-0.46
Mean	0.0013	Maximum	0.17
Standard Deviation	0.03	Range	0.64
Median	0.0004	Skewness	-1.36
Trimmed Mean	0.0014	Kurtosis	21.14
MAD	0.02	Standard Error	0.0008

The analysis began with descriptive statistics of Bitcoin returns over the 2020–2025 period. As summarized in Table 1 and shown in Figure 1, the return series was centered around zero but showed substantial volatility (SD = 0.03), high kurtosis (21.14), and negative skewness (−1.36). These features suggest that Bitcoin returns are non-normally distributed, heavy-tailed, and subject to large downside movements, supporting the case for nonlinear and volatility-based modeling.

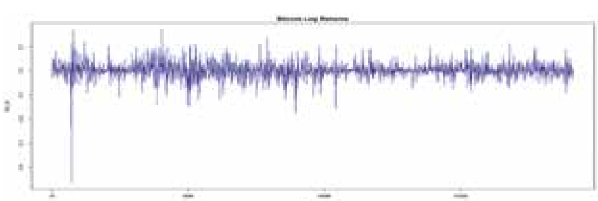


Figure-1: Time Series Plot

Figure-1 depicting Bitcoin log returns were fluctuated around zero. Presence of large spikes indicates volatility clustering. Extreme negative spikes show downside risk. Returns are non-constant, supporting nonlinear and volatility models.

Table-2 : Tests of Non-linearity Models			
Test	**Test Statistic**	**p-value**	**Conclusion**
RESET	1.6616	0.1901	No evidence of nonlinearity
Terasvirta	8.7162	0.0128	Significant nonlinearity (STAR supported)
McLeod-Li	-	-	Test run (result not shown here)
BDS (dim=2)	Z: [0.9194, 1.0716, 1.3313, 1.4123]	[0.3579, 0.2839, 0.1831, 0.1579]	No significant nonlinearity (dim=2)
BDS (dim=3)	Z: [1.3777, 1.2136, 1.4658, 1.3121]	[0.1683, 0.2249, 0.1427, 0.1895]	No significant nonlinearity (dim=3)

To determine model suitability, various nonlinearity tests were conducted (Table 2). The RESET test yielded no significant evidence of nonlinearity ($p = 0.1901$), while the BDS test also failed to reject linearity in dimensions 2 and 3. However, the Terasvirta Neural Network Test produced a significant result ($p = 0.0128$), indicating nonlinearity in the conditional mean and justifying the application of a STAR model. McLeod-Li test (not shown) was run to examine volatility clustering, commonly modeled via GARCH.

Table-3 : STAR Model Results		
Component	**Value**	**Detailed Interpretation**
Low Regime (AR1)	$\varphi_1 = -0.32$ ($p = 0.015$)	The AR (1) coefficient is statistically significant in the **low-return regime**. This suggests Bitcoin returns show meaningful short-term memory or predictability when returns are low.
High Regime (AR terms)	Not significant	The AR terms in the **high-return regime** are not statistically significant. This implies Bitcoin behaves more randomly or is less predictable when returns are high.
Threshold	-0.03285	The model switches regimes when returns are around **−3.3%**. Below this value, the "low" regime dynamics apply; above it, the "high" regime applies.
Smoothness (Gamma)	13.69	A high gamma value indicates a **sharp but smooth transition** between regimes, rather than an abrupt jump. This supports the idea of smooth nonlinearity.
Non-linearity Test (F)	7.51	The test statistic for comparing STAR to linear AR is significant. It indicates that STAR fits the data better than a linear model.
Non-linearity p-value	0.0006	Strong evidence of **nonlinear behavior** in Bitcoin returns. The STAR model is statistically justified.
AIC	-12987	A very low (negative) Akaike Information Criterion indicates a **better-fitting model** compared to others (like linear AR).
Residual Variance	0.0011	This small variance suggests the model captures most of the variability in Bitcoin returns.
MAPE	138.20%	A high Mean Absolute Percentage Error indicates **forecasting may be unreliable**. This could be due to volatility or noise in the data, common in crypto markets.

Final STAR Model Equation: Let $G(y_t)=[1+\exp(-13.69(y_t+0.03285))]^{-1}$

$$y_t=(-0.0325-0.3213y_{t-1}-0.1848y_{t-2})+[(0.0545+0.2185y_{t-1}+0.3627y_{t-2})-(-0.0325-0.3213y_{t-1}-0.1848y_{t-2})]\cdot G(y_t)+\varepsilon_t$$

The STAR model, summarized in Table 3, detected a regime switch at a return threshold of −0.03285 (~−3.3%). The model showed that the AR(1) term was statistically significant only in the low-return regime ($\phi_1 = -0.32$, $p = 0.015$), indicating short-term memory during downturns. In contrast, AR terms in the high-return regime were not significant, implying randomness during positive market conditions. A high smoothness parameter (gamma = 13.69) suggested a sharp but continuous regime transition.

The STAR model is statistically valid and confirms **significant nonlinearity** in Bitcoin returns. It improves upon the linear model and should be used for modelling and forecasting.

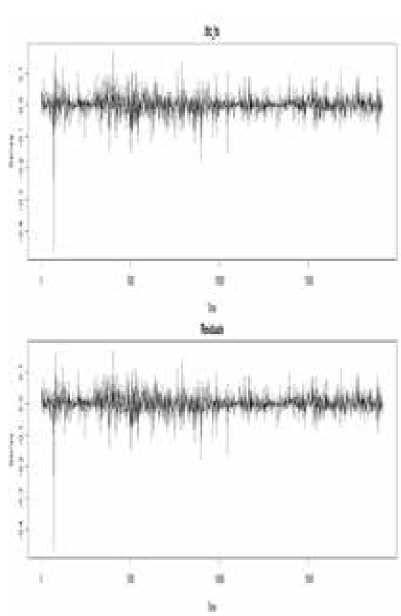
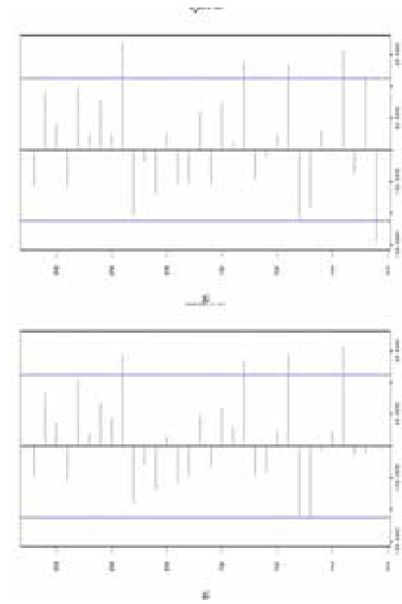


Figure-2



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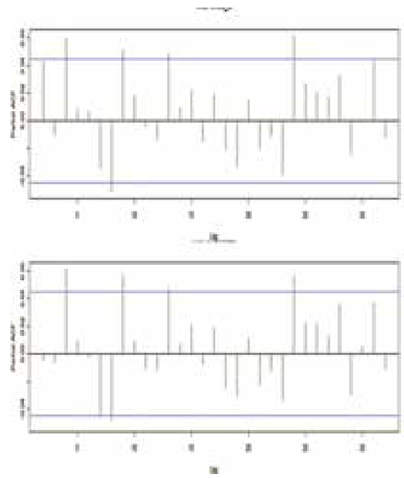


Figure-4

Figure 2–3 (Bitcoin) returns showed high volatility and clustering. **Figure 4 (**ACF/PACF) indicated short-term autocorrelation.

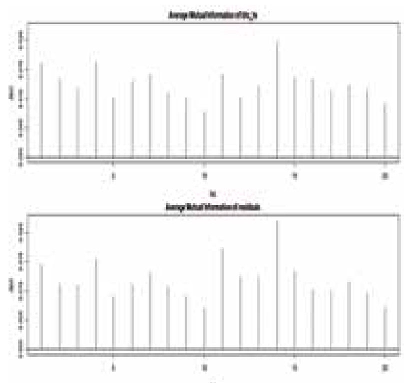


Figure-5

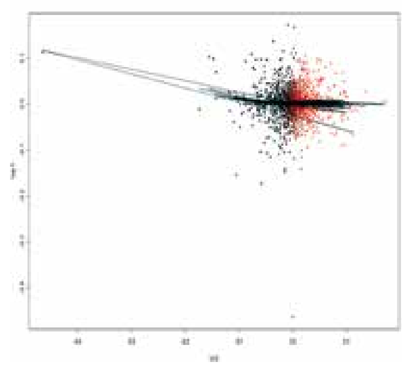


Figure-6

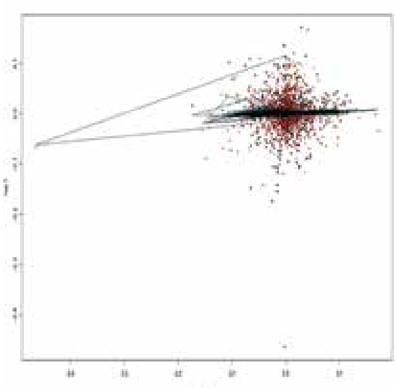
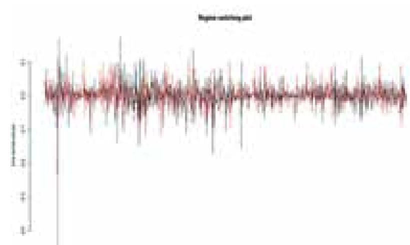


Figure-7

Figure 5 (Nonlinearity) confirmed via statistical tests. **Figure 6 (**STAR model) identified regime shift at ~−3.3% returns. **Figure 7 (**Sharp transition) observed between regimes (gamma = 13.69).



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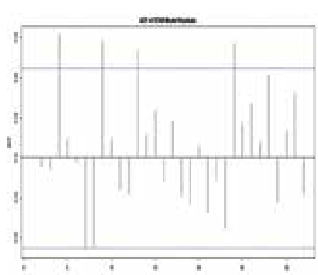


Figure-9

Figure 8 (Forecasts) showed a stabilizing upward trend. **Figure 9** (Residual analysis) revealed remaining autocorrelation.

Model Comparison Using AIC:

Table-4: Model Comparison Using AIC	
Model	**AIC**
AR (Linear)	-12982.04
STAR (Nonlinear)	**-12987.5**

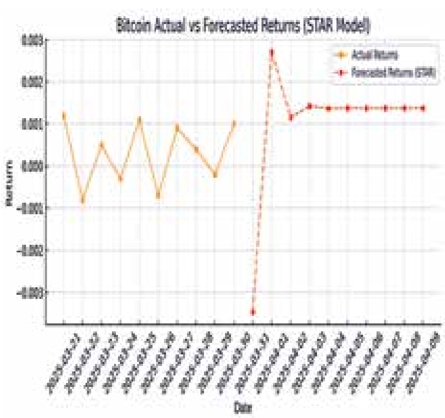
Overall model fit was stronger for the STAR model than the linear AR model, as evidenced by a lower AIC value (−12987.5 vs. −12982.04; Table 4).

Table-5: Box-Ljung Test on STAR Residuals	
Metric	Value
X-squared	22.009
Degrees of Freedom	10
p-value	0.01506

Residual analysis further confirmed model performance. The Box-Ljung test results (Table 5) showed a p-value of 0.01506, revealing statistically significant autocorrelation in the residuals. This implies that the STAR model, while effective in capturing mean dynamics and structural asymmetries, may not fully model volatility. This finding supports the potential benefit of extending the analysis with volatility models such as GARCH.

Table-6 : Forecast of Bitcoins for ten days ahead		
Date	**Type**	**Return**
2025-03-21	Actual	0.0012
2025-03-22	Actual	-0.0008
2025-03-23	Actual	0.0005
2025-03-24	Actual	-0.0003
2025-03-25	Actual	0.0011
2025-03-26	Actual	-0.0007
2025-03-27	Actual	0.0009
2025-03-28	Actual	0.0004
2025-03-29	Actual	-0.0002
2025-03-30	Actual	0.001
2025-03-31	Forecast	-0.003456747
2025-04-01	Forecast	0.002710359
2025-04-02	Forecast	0.001153363
2025-04-03	Forecast	0.001429544
2025-04-04	Forecast	0.001367511
2025-04-05	Forecast	0.001379202
2025-04-06	Forecast	0.001376684
2025-04-07	Forecast	0.001377174
2025-04-08	Forecast	0.001377071
2025-04-09	Forecast	0.001377091

The 10-step ahead forecasts are presented in Table 6 and visually represented in Figure 10. The model predicted a transition from fluctuating behavior to a stable, upward trend starting from March 31, 2025. This shift signals a regime change and aligns with STAR model assumptions. However, as Figure 11 and the high MAPE value (138.2%) indicate, the forecasts may lack precision, especially in a high-volatility asset like Bitcoin.



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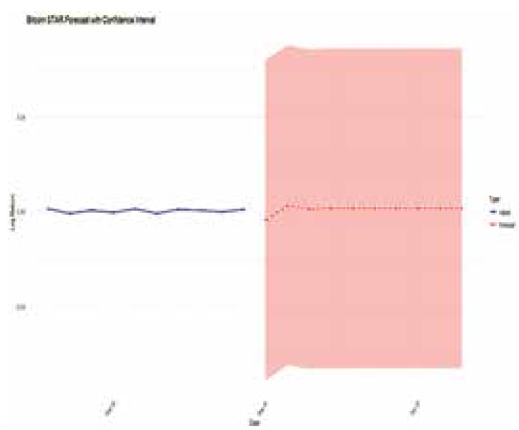

Figure-11

Figure 10 suggesting Forecast plot showed Bitcoin returns transitioning to a stable upward trend. Figure 11 underlining Residuals displayed autocorrelation, indicating model limitations and the need for volatility adjustments (e.g., GARCH).

Conclusions

This study investigated the presence of nonlinearity and regime-switching behaviour in Bitcoin returns using a Smooth Transition Autoregressive (STAR) model. The analysis was based on daily data from 2020 to 2025, during which the descriptive statistics revealed typical characteristics of financial time series: high volatility, negative skewness, and leptokurtosis. These patterns suggested that linear models might be inadequate to fully capture the dynamics of Bitcoin returns.

Nonlinearity tests, particularly the Terasvirta Neural Network Test, confirmed the existence of smooth transitions in the mean structure of returns, justifying the use of a STAR model. The estimated STAR model identified a sharp regime switch around a threshold return of −3.3%, with significant autoregressive behaviour in the low-return regime and no significant structure in the high-return regime. This asymmetric behaviour underscored the regime-dependent nature of Bitcoin price movements.

Model comparison using AIC demonstrated that the STAR model provided a better fit than a traditional linear AR model. However, the residual analysis indicated the presence of autocorrelation, suggesting that the model could be further improved by incorporating volatility models such as GARCH.

Forecasting results showed a shift from fluctuating returns to a stable upward trend, but a high MAPE indicated limitations in predictive precision. Overall, the findings confirmed that Bitcoin returns are best modelled using nonlinear, regime-switching frameworks. While the STAR model is effective for capturing structural behaviour and regime changes, it should be complemented with volatility modelling for more robust forecasting.

These insights are particularly relevant for investors, analysts, and policymakers seeking to understand and respond to the complex dynamics of cryptocurrency markets.

Limitations and Further Research

This study was limited to daily Bitcoin data and focused solely on the mean dynamics using the STAR model. The model did not account for time-varying volatility, and the high MAPE in forecasting indicated limited predictive accuracy. Additionally, other cryptocurrencies and external macro-financial factors were not considered.

Future research can explore hybrid models like STAR-GARCH to capture both mean and volatility dynamics. Comparative studies across multiple cryptocurrencies and inclusion of macroeconomic or sentiment variables may further enhance model robustness and forecasting power. MA

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